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ISSN 1751-3030


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# A Mathematical Study of a Symbol: the Vesica Piscis of Sacred Geometry

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Published in matho.philica.com

## Abstract

In this paper, we are proposing a study of the Vesica Piscis, a symbol of Sacred Geometry, starting from mathematics. This study aims to give a deeper comprehension of the hidden meanings of this icon. The paper is also showing that scientific and philosophical studies can be integrated, leading to very interesting results.

## Article body

**A Mathematical Study of a Symbol: the Vesica Piscis of Sacred Geometry**

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**Abstract**: In this paper, we are proposing a study of the Vesica Piscis, a symbol of Sacred Geometry, starting from mathematics. This study aims to give a deeper comprehension of the hidden meanings of this icon. The paper is also showing that scientific and philosophical studies can be integrated, leading to very interesting results.

**Keywords**: Sacred Geometry, Geometry, Architecture.

**Introduction:** The Vesica Piscis is one of the most important symbols of the Sacred Geometry, a discipline ascribing symbolic meanings to geometric shapes and proportions, which has its roots in the study of nature and natural principles [1,2]. This Geometry is representing numbers in the tridimensional space, differently from the Euclidean geometry, because the concepts involved have symbolic values, used with the aim of facilitate "the evolution of the soul" [3]. In this manner, the symbols of the Sacred Geometry assume a peculiarity: they silently appear in several places, like in churches and temples, and in some cases, are ruling architectures. For example, the Vesica Piscis appears in the design of Bernini's oval part of Saint Peter's Square in Rome [4], the most important place of the Catholic Church.

Also in the case of Euclidean geometry, we have to consider that some geometric diagrams can play a crucial role in visualizing mathematical proofs, and therefore in the evolution of our knowledge of the world. In [5], twenty of these icons of mathematics are presented and discussed. As proposed in this reference, each icon has a presence in real life and a primary mathematical characteristic, and it gives origin, with its visual proofs, to a wide range of mathematical facts. And then, from them, we can find the classical results of plane geometry, properties of numbers, trigonometric relations identities, theorems of mathematics, and so on. If we can from icons discover mathematics, we can also try to find symbolic meanings of the icons through the use of mathematical techniques, as we will see in the following discussion. In this paper, the method will be applied to the Vesica Piscis. In the following, we will introduce the symbol, then the mathematical approach to it and role in arts and architecture, before the disclosure of the hidden meaning of the symbol.

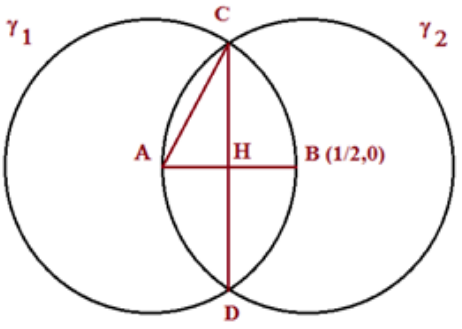


Figure 1: The Vesica Piscis formed by two identical circumferences.

**The symbol**: The Vesica Piscis, the "bladder of a fish" in Latin, is shown in the Figure 1. This symbol is created by two circumferences, $\gamma_1$ and $\gamma_2$, having a radius of the same length. The center of each circumference lies on the other circumference. The overlapping area of the two circles is called "eye" or "mandorla", almond in Italian. In Figure 1, the two centers are denoted by letters A and B. The Vesica Piscis also includes segment AB and its perpendicular segment CD, having midpoint H . Without loss of generality, we can assume the radius r of both circumferences be equal of the unit length. In fact, if r = $a$, with $a > 0$ and $a \neq 1$, then the resulting symbol is a homotetic transformation of the same symbol with radius 1. Thus, the shape of the symbol does not depend on the given radius.

**Mathematical study**: Starting from the two circumferences $\gamma_1$ and $\gamma_2$, we want to compute lengths AB and CD of the respective segments. Let $r_1$ and $r_2$ denote respectively the radii of circumferences $\gamma_1$ and $\gamma_2$ , we have $r_1 = r_2 = 1$. Segment AB is, by construction, the radius of both circumferences, and then its length is equal to unit. Since H is the midpoint of segment AB, the segment AH has a length equal to 1/2. By construction, AC is a radius of circumference $\gamma_1$ , and the its length is equal to 1. Triangle ABC is equilateral. Triangle AHC is rectangular, being segment CH perpendicular to AB. Therefore, we can apply Pythagoras' theorem to triangle AHC and compute the length of segment CH. We have:

$$(1) \qquad \overline{CH} = \sqrt{\overline{AC}^2 - \overline{AH}^2} = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$$

By the symmetry of the figure:

$$(2) \qquad \overline{CD} = 2\,\overline{CH} = 2\,\frac{\sqrt{3}}{2} = \sqrt{3}$$

The ratio of the two main segments is:

$$(3) \qquad \frac{\overline{CD}}{\overline{AB}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

We have then, geometrically, the square root of 3, which is an irrational number.

However, since we referred to Pythagoras' theorem, let us consider (3), written as:

$$(4) \qquad \frac{\overline{CD}^2}{\overline{AB}^2} = 3$$

And, for this equation, Archimedes reported the value $(1351/780)^2 > 3 > (265/153)^2$ [6].

The square root of 3 is also known as Theodorus' constant, named after Theodorus of Cyrene [7]. However, for what concerns the icon in the Figure 1, to the square root of 3 we have to add other numbers: as shown in [8], we can find the square root of 2, the square root of 5 and the Golden Ratio. However, it is number 3, the relevant one of this icon.

**The icon in architecture and arts:** In the introduction, we have already mentioned that the Vesica Piscis is appearing in the layout of the oval part of Saint Peter's Square, a square planned by Gian Lorenzo Bernini (1598-1680) [9,4] (see Figure 2). The great artist and architect had probably followed the canons, in his planning of that part of the Square framed by an "ovato tondo". This figure was the approximation of ellipse used in architecture. The Vesica Piscis was recommended by Sebastiano Serlio (1475-c.1554), the Italian Mannerist architect that helped canonize the classical orders of architecture. This construction of "ovato tondo", besides being the simplest and quickest manner to construct oval spaces, was also suggested for its beauty. In fact, it became the standard ellipse approximation used in architectural practise [10]. Let us note that the Vesica Piscis was already used as a proportioning system of Gothic architecture, as illustrated by Cesare Cesariano (1475-1543), in the first Italian-language version of Vitruvius' De Architectura [11]. Cesariano called it "the rule of the German architects". This geometric shape was used frequently in Gothic cathedrals then, and in fact, the construction appears in the Portfolio of Villard de Honnecourt, a

13th-century artist from Picardy in northern France [12].

The Vesica Piscis was known to Pythagoreans as the "Potential Logos", symbolizing the Dyad that becomes a Triad, the harmony or Logos [13]. Later, this symbol migrated to the Christianity [14]. In the Christian art, the Vesica Piscis often became the halos of light, that is, the aureoles surrounding God, Jesus Christ, Virgin Mary and Saints (see an example in the Figure 2). Also the seals of ecclesiastical organizations had this icon as a frame. The Vesica Piscis has been used as a symbol within Freemasonry too, and the proper shape for the enclosure of the seals of Masonic lodges [15].

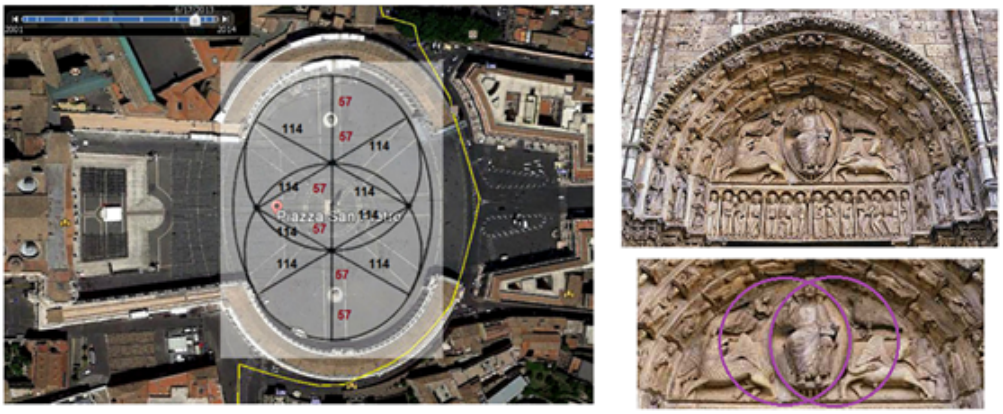


Figure 2: On the left, the "ovato tondo" of Bernini's Square in front of Saint Peter's Basilica, seen in a Google Earth image. To evidence the geometry of this ovato, black and red numbers give the lengths (in pixels) of the corresponding lines. The obelisk is the center of a Vesica Piscis. On the right, we see a Vesica Piscis in the central tympanum of the Royal portal of the Gothic cathedral of Chartres.

**The hidden meaning of the icon**: After the examples of Vesica Piscis given in the previous section, let us consider again the results we have obtained from calculus in Eqs.(1)-(4). In the Sacred Geometry, God, or the One of the Universe (Unus mundi, in Latin) or the divinity, is represented as a circle with radius 1, because One is the concept of unity, being God omnipresent and the creator of the world. In the Vesica Piscis, the projection of the same circumference starting from the initial one, given the concept of Duality. In a number of spiritual believes, a soul temporary separates from the One (Unity) to know herself. Therefore, we have a spiritual journey of the soul from Unity to Duality and then back to Unity, so that a highest level of evolution is acquired. However, the two segments AB and CD form a cross. This cross represents Christ and the concept of Trinity, as witnessed by the ratio of the squared lengths of these segments, which is number 3, i.e., the number of Trinity. In fact, as we have already mentioned, the Vesica Piscis migrated from the Pythagorean philosophy, where it was a Triad, to the Christianity becoming a Trinity [16].

**Conclusions**: In this paper, we have proposed a study of Vesica Piscis, starting from mathematics. This study is fundamental for a deeper comprehension of hidden meanings of this icon, revealing at least two important considerations. First, in the discipline of Sacred Geometry, geometry and calculus are not only human and artificial inventions, but are symbolic means through which the divine extends its process of Creation and Life. Second, in a scholar approach, this work proves how scientific and philosophical studies can be integrated, leading to very interesting results.

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## Information about this Article

This Article was published on 23rd January, 2016 at 14:09:04 and has been viewed 16 times.